EFFECT ON FREQUENCY FOR ADDING MASS OR STIFFNESS

WILLIAM C. FOILES

1. INTRODUCTION

This article discusses approximation techniques that compute the change in required mass or stiffness addition for a desired frequency change. The methods assume that the changes to mass or stiffness do not significantly alter the original modeshapes.

Some structural modifications may inadvertently change or may directly attempt to change the mode shape, and thus this method would not reliably compute the effect on natural frequency.

2. MASS ADDITION TO A STRUCTURE

An approach to mass addition would be to consider this as an energy problem. The assumption is that the mass doesn’t (substantially) change the mode shape — if it does this will not work. Thus, to some degree this is for ‘small’ system perturbations. Also, all mass additions occur at the same point or points in the same ratio at these points for the trial and final weight addition.

An advantage to this method is that the resulting formula will not need unit conversion when using weights instead of masses.

\[ f_1 \] - original frequency
\[ f_2 \] - frequency after mass addition
\[ w_a \] - Trial weight addition, yields \( f_2 \)
\[ w_d \] - Weight addition, yields \( f_d \)
\[ f_d \] - Desired frequency with final mass addition
\[ m_a \] - mass addition to get \( f_2 \), may be at more than one location
\[ m_d \] - desired mass add or required mass add \((w_d \text{ and } w_a \text{ are the same weights instead of masses})\)

\( K \) - Stiffness Matrix
\( M \) - Masss Matrix
\( x \) - vector, generally a modeshape

A Rayleigh’s equation (quotient) with a modeshape gives,
When using a modal vector $x$ in the above Rayleigh quotient, $f_1$ has the value corresponding to the same natural frequency. Note certain other technical assumptions have been made to establish this consequence.

With the mass addition,

$$f_2^2 = \frac{x^T K x}{x^T (M + m_a) x}$$

(2)

Mass addition has a negative effect on frequency. This can be seen below.

$$\frac{x^T m_a x}{x^T K x} = \frac{1}{f_2^2} - \frac{1}{f_1^2}$$

(3)

and

$$\frac{x^T m_d x}{x^T K x} = \frac{1}{f_d^2} - \frac{1}{f_1^2}$$

(4)

If the mode shapes do not change then mass addition required to produce the desired frequency effect can be scaled from the trial mass addition. This includes the possibility that the trial mass addition had a number of locations and that $m_a$ and $m_d$ have the structure of matrices with more than one element. Remember that $x^T m_a x$ and $x^T m_d x$ are scalars, and multiplying each element of $m_a$ by a scalar results in each of the previous scalar values to be multiplied by the same scalar. The result is:

$$m_d = \frac{1}{f_d^2} - \frac{1}{f_1^2} m_a$$

(5)

V Because of the ratios weights or mass can be used, Hz, rpm, or rad/sec can be used for the frequencies in the above formula, also. The conversion factors cancel.

Example

$$f_1 = 19\text{Hz}$$
$$f_2 = 15\text{Hz}$$
$$f_d = 10\text{Hz}$$
To find the desired weight for the example above, with initial weight addition = 10 lbs.:

$$w_d = \frac{1/10^2 - 1/19^2}{1/15^2 - 1/19^2} \times 10\text{lbs}$$

$$= 43.18\text{lbs}$$

Such a large frequency change may violate the assumption concerning alteration of the mode shape.

3. **Stiffness Addition to Structure**

Adding stiffness may move up frequencies, but stiffness additions have accompanying mass, which implies additional resonances and potential loss of effective stiffness addition. Many times, mass additions can effectively act as mass, because any internal resonance to the mass element would have a very high frequency and minimal stiffening to the base structure occurs.

Stiffness additions may occur at several physical locations, as may mass additions. When calculating the final stiffness additions, all locations require proportionate scaling.

Assumptions:

(1) Pure stiffness addition, i.e. no mass effects - a massless spring element or elements
(2) Mode shape does not change with stiffness addition
(3) Final stiffness added in same proportions as original modification

This will solve for a relative stiffness to the original.

- $k_a$ - original stiffness addition
- $k_r$ - required stiffness addition to meet goal frequency

Rayleigh’s equation (quotient):

$$f_1^2 = \frac{x^T K x}{x^T M x} \quad (6)$$

Similarly,

$$f_2^2 = \frac{x^T (K + k_a) x}{x^T M x} \quad (7)$$

Thus,

$$\frac{x^T k_a x}{x^T M x} = f_2^2 - f_1^2 \quad (8)$$
and

\[
\frac{x^T k_rx}{x^T Mx} = f_d^2 - f_1^2 \tag{9}
\]

Divide these equations to obtain the following.

\[
\frac{x^T k_rx}{x^T k_ax} = \frac{f_d^2 - f_1^2}{f_2^2 - f_1^2}
\]

\[k_r\] can be obtained by scaling \(k_a\) as follows:

\[
k_r = \frac{f_d^2 - f_1^2}{f_2^2 - f_1^2} k_a \tag{10}
\]

As before unit conversions will cancel for the frequency and stiffness because of the ratios.